

Multi-Attribute P-Median Model for Location of Back-up Transformers

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Abstract

Deciding on where to place a back-up transformer is not an easy task. It involves factors that might be considered irrelevant or of little influence, but which in fact are extremely important and ensure the success of the location process. This procedure requires close attention to key criteria as well as careful planning and lengthy, widely-ranging studies of placement alternatives before definitively choosing where the back-up transformer should be placed. This study contributes a multicriteria perspective on the question, by proposing a methodology which takes the key criteria into full when determining the best location for a back-up transformer. These include considerations of public health, the spread and location of local industries, and the population of the area, as well as drawing up an index of the distance of the backup transformer from the population which is usually deemed as a transport cost. When these criteria are taken into consideration, the best location is that which generates the least loss in maximizing the total utility. This can be interpreted as the judgment made regarding the location of the back-up in a substation, taking into consideration the benefits of the back-up being in this place.

Keywords: Location of transformers, Location of Equipment, Multi-criteria decision, *MAUT*

Introduction

A transformer is an electrical device the purpose of which is to transfer electrical energy from one circuit to another through inductively coupled conductors. A varying current in the first or primary coil creates a varying magnetic flux in the core of the transformer, and thus a varying magnetic field through the secondary coil.



In electric power distribution systems, transformers are characterised as having a high useful life and costs. These factors become a complex problem when deciding on the number and location of back-up transformers in network systems in order to get satisfactory levels of system availability.

Location theory is an area with an extensive literature and is applied in several industrial sectors (Hakimi, 1983). There are a number of approaches that can be used to determine the best location, whether for businesses, equipment, real estate or any device with low mobility or for which long-term permanence in a particular place is made necessary due to the high investments associated with setting up there or for strategic reasons (Almeida *et al*, 2006), (Owen & Daskin, 1998), (Thizy, 1993), (Mirchandani & Francis, 1990), (Robeson *et al*, 1994), (Bowersox, 1978) and (Almeida, 2010).

According to Farahani *et al.* (2010), where to locate a facility can be considered a one-hundred year old science. Several researchers studied facility location a long time ago, the publication of Alfred Weber (1909) being regarded as the most important milestone in the history of location science.

Where to locate back-up transformers aims at determining the optimal location of these technical reserve facilities in a network power distribution system, taking into account factors of cost and service level.

In the particular case under study, we consider back-up equipment designed to enter operation when a similar piece of equipment exercising the same function fails. There are usually a number of places which can serve as back-up locations for reserve equipment. In this case, the available localities are power distribution substations. The problem is to determine a strategic location for back-up transformers among those available, while avoiding negative impacts and taking the following factors into account:

- The distance between substations (representing more than just the cost of displacement as this also involves how much time the users will go without service);
- The index that represents the number and size of industries served by the substation;
- The size of the population served by the substation;
- The index that represents the number and size of health services in the area served by the substation.

Industrial Location and its most Important Factors

A. Facility Location

Regardless of the type of activity, whether commercial or industrial, decisions on the location of facilities play a strategic role in an organization.



Since it is directly linked to planning, determining the location of a facility should take into consideration all the positive and negative aspects inherent to the possible locality chosen (Bowersox, 1978), (Hakimi, 1964) and (Chase & Aquilano, 1989).

Some of the factors that can influence location decisions are addressed below, the majority of these being related to the siting of the stock of raw materials. Some of the reasons why firms locate next to the sources of raw material area as follows:

- The perishability of materials: for example, food industries are very often situated near regions that supply the material;
- Transport Costs: activities for which the raw materials are of high volume and low value, an example of which is the cement industry, justify location next to these deposits. When companies work with suppliers in various locations, then they have to seek other ways to minimize the cost of transportation;
- Labor: cultural aspects in certain regions, such as a reputation for high absenteeism and personnel turnover, can lead companies to opt for one place over another;

Other factors such as the location of consumer markets or water and electricity supplies may also affect the choice of location.

B. Evaluation of alternatives

In this section, some of the fundamental models for different problems of location will be briefly reviewed. More detailed descriptions can be found in Chase & Aquilano (1989), Bowersox & Closs (1996) and Lambert & Stock (1992).

1) Qualitative Deliberation

This is used when local costs are not available. Each critical factor is arbitrarily assigned a value. Usually, the values are weighted according to their importance and a total is derived for each location, thus providing a sum which can be compared among the potential locations. However, this weights represent scaling constants of an additive model and should be properly elicited (Almeida, 2010). The location with the highest sum will be chosen.

$$Ni = \sum_{j=1}^{k} F_{ij} P_j \tag{1}$$

K factors

F_{ii}: value of factor "j" at locality "i"

P_i: weight or scale constant of factor j



2) Comparison between variable and fixed costs

With respect to the location of non-profit companies or institutions, the ideal is to establish the costs incurred in the choice of each locality. Put simply, the costs can be divided into those that are fixed and variable, for later analysis of the balance between them, which can be determined in the following manner:

When the quantity required is known, it is possible to calculate the profit associated with each place, choosing the one that will yield the most profit. If the revenue does not depend on the locality considered, the overall cost of each locality should be calculated, the one with the lowest total cost being opted for.

The equilibrium point of each locality is calculated and the one with the lowest equilibrium point is chosen.

3) Dimensional Analysis

Dimensional analysis is used when there are alternatives with qualitative factors and some quantified costs.

Consider the qualitative factors according to a scale of their relative values:

Assign weights to the qualitative and quantitative factors.

K factors

F_{ii}: value of factor "j" at locality "i"

P_i: relative weight of factor j

The coefficient of merit for location 1 in relation to location 2 is given as:

$$CM_{1,2} = \left(F_{1,1} / F_{2,1}\right)^{p_1} \left(F_{1,2} / F_{2,2}\right)^{p_2} \dots \left(F_{1,k} / F_{2,k}\right)^{pk}.$$
(2)

If the qualitative factors are placed on a scale, the lowest values correspond to the greatest benefits. Therefore, if $CM_{1,2} < 1$, location 1 should be chosen. Conversely, if $CM_{1,2} > 1$, location 2 should be chosen.

Note that if the scale of qualitative factors is weighted so that the highest numbers correspond to the greatest benefits, the power of these factors should be inverted (negative power).

This model is a particular case of the multiplicative model for multicriteria aggregation (Almeida, 2010)

4) Center of Gravity Model

This is used to locate a new facility within a network of existing installations or markets, in order to minimize transport costs.

The center of gravity is the place under consideration representing the minimum distance from the other installations or markets.



With respect to the circulation of merchandise, the center of gravity model considers distances by the volume of the quantity of merchandise. In the case of an electricity generating station, a weight might be attributed to the relative demands of the region served. Generally this weight is related to the size of the population served by the new installation.

The horizontal and vertical coordinates are determined for each facility or market by using a map and a system of orthogonal axes.

To determine the center of gravity (coordinates G_x and G_y), the following equations are used:

$$G_x = \sum d_{ix} p_i C_i / (\sum p_i C_i).$$
(3)

$$G_{y} = \sum d_{iy} p_{i} C_{i} / (\sum p_{i} C_{i}).$$

$$\tag{4}$$

Where:

d_{ix} is the horizontal coordinate of the market

d_{iv} is the vertical coordinate of the market.

p_i is the transport cost in the direction of installation "i"

C_i is the volume transported from/to installation "i"

If the transport costs are equal in all directions, we have:

$$G_x = \sum d_{ix} p_i C_i / (\sum C_i).$$
⁽⁵⁾

$$G_{y} = \sum d_{iY} p_{i} C_{i} / (\sum C_{i}).$$
(6)

5) Median Model

This has the same objectives as the previous model. An important factor in this model is that the load moves only horizontally or vertically. The basic data of the model are: the vertical and horizontal coordinates of the existing installations and the load that needs to be moved to/from each of the installations. After determining the coordinates of the location sought, the overall transport cost is calculated as follows:

$$CT = \sum C_i P_i (d_{ix} + d_{iy}). \tag{7}$$

6) Location of emergency services

With regard to the location of emergency services, such as fire brigade and ambulance stations, speed is fundamental and the principal factor for consideration.



The following steps are used to determine a location (Ardalan, 1988):

- 1. Define the coverage area;
- 2. Define potential locations for the emergency service stations (generally in one of the neighborhoods of the area);
- 3. Identify the connecting routes and travel time from one area to another;
- 4. Design a matrix based on the minimum travel time from one area to another;
- 5. Identify the maximum access time from one area to another (for each of the areas);
- 6. Among the maximum times, select the area which has the least access time.

The problem of locating back-up transformers is similar to that of locating emergency services such as ambulances, fire brigade stations and so on.

The Suggested Model

In this section, the building of a multi-attribute P-median model to determine back-up transformers location is presented. Classical facility location models like the P-median problem (PMP) and the uncapacitated fixed-charge location problem (UFLP) implicitly assume that, once constructed, the facilities chosen will always operate as planned. However, facilities "fail" from time to time due to poor weather, labor actions, changes of ownership, or other factors (Snyder & Daskin, 2005).

The paper by Leung & Khator (1995) deals with the issue of acquiring and relocating substation transformers in power systems. Their study is designed for one specific context: that of the Florida Power and Light (FPL) process within the overall planning of transformers. This process involves three decisions: capacity planning; acquisition and scheduling of processors; and implementing the plan. The object of study focuses on the second activity that includes the study of re-lease. The model adopted to solve the problem is based on mixed 1-0 linear programming. The planning process uses the technique of reserving resources so as to meet the general planning requirements for acquiring processors in the system, besides the acquisition itself, in view of the deadlines for system expansion. Actually, this proposal should not be easily accepted by the maintenance area, since the availability of the system in operation may be affected. Perhaps this aspect explains the fact recorded in that article that a combined study of acquisition and relocation of transformers was not found in the literature. The literature cited in the article is related to the area of designing power distribution systems. It was also noted that the model does not incorporate modeling the reliability of a system in operation.

Planning the operation of the response to the need for the emergency distribution of electricity involves a host of decision problems that can be modeled and solved using operations research methodologies. The importance of these problems is obvious from the impact that fault situations have on customers and electric utilities. Fault situations may cause "in extremis" states where service is interrupted



in distribution systems, thus reducing the quality of service and causing financial losses for electric utilities. These losses are difficult to quantify monetarily but can be significant in specific situations (Perrier *et al*, 2010).

There are some approaches to the problems of logistics and resource management within the electricity sector that may be related to the study in focus, amongst which the following papers stand out: Costa and Silva (2009) show a probabilistic methodology based on stochastic process theory to obtain the optimal number of back-up transformers in a power system; Xu et al. (2008) applied the Fuzzy Analytic Hierarchy Process (FAHP) to select the location for a transformer substation: Zambon et al. (2005), present an interesting article addressing the context of planning the expansion of the electricity sector, in which the problem of finding a location for a thermoelectric plant (TPP) to be installed, is tackled by combining the tool of a geographical information system (GIS) with multicriteria decision methods. The paper by Cunha et al. (2004) addresses how and where to locate a transformer in rural properties by using the center of gravity model. Wang *et al.* (2004) describe a location model using a double-phase heuristics that is applied to manage the resources for restoring electrical installations after a break in supply. Khodr et al. (2003) propose a probabilistic method that is designed to support the planning of electric power systems in selecting locations for power distribution substations, taking into account the daily cycle of loads.

Finding the best location for back-up supply stations is equivalent to using the traditional models of business repositioning, with the back-up transformers representing the businesses: in this case the available places are substations with comparable supplies and may be considered sources of supply in that, when located in a substation, the back-up can meet the demand of a failure in a substation nearby.

The multi-attribute P-median model proposed is based on three criteria. Factors such as the size of the population (pop_i) , the degree of industrialization (ind_i) and the extent of health services (hs_i) are considered. The distance factor (d_{ij}) represents the distance between substations *i* and j = 1, ..., ns, where *ns* is the number of substations. The distance factor works as a multiplier weight in relation to *pop*, *ind* and *hs*.

Some aspects of the problem of context are considered in building a multicriteria model based on a multi-attribute utility function, such as: the question of choice, the basic framework of underlying preferences, and the means of support to a multi-criteria decision (Gomes *et al.*, 2009).

Based on our studies of preference, three factors were seen to have similar behavior when they deal with a negative exponential utility function. This means that the greater the product of population x distance, the utility of a particular location falls exponentially rather than linearly. This behavior can be understood as follows: as the magnitude of the indicator corresponding to the criterion increases, the effects on the value of a consequence decreases, independently of the criteria. So the first few hours following the interruption of power service are the most critical and, after a certain level of customers is affected, the utility of consequence hardly varies.

Thus, we have:

In Eq. (8), the general form of the negative exponential function is characterized.

$$U(z) = \exp(-A \cdot z) \tag{8}$$

Where: the parameter A is a constant of the negative exponential function based on the assumption that $U(Max(z) \approx 0$.

In Eq. (9), the matrix form of the negative exponential function is represented, taking into account the distance factor d_{ii} .

$$U(y_{ij}) = \exp(-A_y \cdot y_i \cdot d_{ij})$$
⁽⁹⁾

In Eq. (10), Eq. (11) and Eq. (12), the one-dimensional matrix utility functions for the criteria population (pop_i) , the degree of industrialization (ind_i) and the extent of health services (hs_i) are shown.

$$U(pop)_{ij} = \exp(-A_1 \cdot pop_i \cdot d_{ij}), \ \forall \ i, j = 1, ..., ns$$
(10)

$$U(ind)_{ij} = \exp(-A_2 \cdot ind_i \cdot d_{ij}), \ \forall \ i, j = 1, ..., ns$$

$$\tag{11}$$

$$U(hs)_{ij} = \exp(-A_3 \cdot hs_i \cdot d_{ij}), \ \forall \ i, j = 1, ..., ns$$

$$(12)$$

Where: A_1 , $A_2 \in A_3$ are constants to be estimated in order to satisfy the assumption that $U(Max(z) \approx 0$.

The multi-attribute P-median model corresponds to the following multiattribute binary programming:

$$\begin{aligned} Max \ \sum_{i=1}^{ns} \sum_{j=1}^{ns} (K_1 \cdot U(pop)_{ij} + K_2 \cdot U(ind)_{ij} + K_3 \cdot U(hs)_{ij} + K \cdot U(pop)_{ij} \cdot U(hs)_{ij}) \cdot x_{ij} \\ subject to \ \sum_{i=1}^{ns} x_{ij} = 1; \ j \in N \\ \sum_{j=1}^{ns} x_{jj} = nb \\ x_{ij} \le x_{jj}; \ i, j \in N \\ x_{ij} \in \{0,1\}; \ i, j \in N \end{aligned}$$
(13)

Where:

 K_{p}, K_{2}, K_{3} , and K are scale constants related to the respective attributes; N is a set of substations, $N = \{1, ..., ns\}$;

$$D = [d_{ij}]_{ns \times ns}$$
 is the distance matrix, with $d_{ii} = 0, i \in N$;



nb is the number of back-up transformers;

 pop_i = the size of population served by the substation *i*

 ind_i = the degree of industrialization served by the substation i

 hs_i = the extent of health services served by the substation *i*

 x_{ij} is a decision matrix variable, where $x_{ij} = 1$ if the backup transformer of the substation *i* is designated to substation *j*, and $x_{ij} = 0$, otherwise. And $x_{jj}=1$ if the substation *j* is designated to keep a back-up transformer (a median) and $x_{jj}=0$, otherwise where $i, j \in N$ ($i \neq j$)

The parameters A_p , $A_{2'}$ and A_3 should be established by the decisionmaker using his/her preferences regarding these attributes. The authors of this article investigated the preferences of the manager of the project under study, as per the procedures for assessing preferences laid down by Keeney & Raiffa (1976).

When the aggregation of the three factors is examined, an independence among the attributes is found, except between the health service and population criteria. That is, preferences with respect to health may be influenced by a difference in the size of the population. The model which represents these conditions corresponds to a multi-linear model expressed in the objective function Eq. (13):

The objective function represents the utility for substation k, if the back-up is located in substation j. In order to have an indicator for the location of the back-up in substation j, the sums of the utilities of location for all the substations k should be calculated.

The maximum utility found for all substations recommends the locality for the back-up transformer.

Case Study from CELPE Using a Computer Tool

In this case-study, the numerical application focuses on how to decide which substation should host a back-up transformer of CELPE – The Energy Company for the state of Pernambuco. 19 options were considered of substations with equipment compatible with the back-up unit which would be installed.

These substations belong to a network located in a narrow belt in the state of Pernambuco, a zone which is called the Agreste. This area is shown in Figure 1.



Figure 1 – Agreste of Pernambuco

Table 1 - Population	, Health and	Industrialization	factors of the	areas served by	the substations
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Substations	Population (inhabitants)	Health service units	GDP (000's Brazilian Reais)				
Caruaru	298,501	137	1,993,295				
Garanhuns	131,313	66	742,593				
Santa Cruz do Capibaribe	80,330	18	332,112				
Gravatá	75,229	30	306,637				
Belo Jardim	74,028	26	504,735				
Pesqueira	64,454	33	236,259				
Bezerros	58,354	24	232,859				
Limoeiro	57,243	45	219,496				
Surubim	56,795	26	205,142				
Buíque	53,272	16	172,447				
São Bento do Una	49,372	13	208,020				
Bom Conselho	45,250	16	150,992				
Brejo da Madre de Deus	42,250	19	125,475				
Bom Jardim	40,924	29	117,505				
Bonito	40,832	20	139,985				
Águas Belas	39,672	9	115,899				
São Caitano	36,336	14	102,243				
Lajedo	34,809	17	138,826				
Toritama	33,206	9	122,928				

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The 19 towns cities with highest populations in the Agreste were considered. The data on the population (pop_i) were collected from the IBGE (2009). The degree of industrialization (ind_i) of the areas served by the substations was obtained from the Gross Domestic Product of the towns and the health services indicator (hs_i) was obtained by listing the number of health service units. These data are given in Table 1.

Distance factors (d_{ij}) of these cities are shown in Table 2, based on the estimated distance of these towns from each other obtained through Google maps API.

The location of these substations are shown in Figure 2, a partial map of Pernambuco.

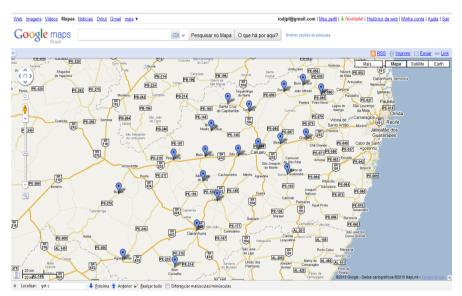


Figure 2 - The locations of the Agreste substations on a partial map of Pernambuco

When building the one-dimensional utility functions, parameters of exponential functions were determined by the property that the utility of the maximum value obtained from one dimension when this is equal to 0.01 (U(max(X))=0.01). For example, in terms of the population attribute, the maximum value of the matrix $pop_i d_{ij}$ was 54,924,184 km.inhabitant. Assuming that U(54,924,184) = 0.01, we have $exp(-A1.\ 54,924,184) = 0.01$. Then, $A_j = -ln(0.01)/54,924,184$. The same process was used for the other two attributes. The one-dimensional utility function of population attribute is shown in Figure 3.

Table 2 - Distance factors of substations (kilometers)



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Toritama	38.7	136	23.2	89.7	87.4	117	67.1	82.3	40	182	112	184	54.4	70.5	91.8	217	55.5	100	0
Lajedo	67	39.2	122	115	62	69.5	91.9	148	140	148	21.3	87.7	129	144	117	120	44.6	0	100
São Caitano	22.4	80	7.7 <i>.</i>	70	32	61.9	47.4	103	95.9	127	56	129	84	99.4	72.1	161	0	44.6	55.5
Águas Belas	184	84.2	239	231	152	107	208	264	257	97.8	125	62.4	245	261	206	0	161	120	217
Bonito	57.9	125	113	50.8	104	134	28.5	100	99.2	199	128	174	120	96.4	0	206	72.1	117	91.8
Bom Jardim	78.8	179	93.6	79.4	131	161	69.2	29.9	30.5	226	155	228	125	0	96.4	261	99.4	144	70.5
Brejo da Madre de Deus	66.8	164	57.4	118	116	145	95.2	137	94.4	211	140	212	0	125	120	245	84	129	54.4
Bom Conselho	150	46.5	206	198	142	136	175	231	224	156	102	0	212	228	174	62.4	129	87.7	184
São Bento do Una	78.5	56.7	134	126	40.7	48.2	103	159	152	113	0	102	140	155	128	125	56	21.3	112
Buíque	149	111	205	197	112	66.3	174	230	223	0	113	156	211	226	199	97.8	127	148	182
Surubim	78.7	175	63.2	73.9	127	157	72	42.3	0	223	152	224	94.4	30.5	99.2	257	95.9	140	40
Limoeiro	82.3	183	106	49.6	135	164	72.8	0	42.3	230	159	231	137	29.9	100	264	103	148	82.3
Bezerros	33.2	127	88.5	26.1	79.3	109	0	72.8	72	174	103	175	95.2	69.2	28.5	208	47.4	91.9	67.1
Pesqueira	84.3	91.4	140	132	46.5	0	109	164	157	66.3	48.2	136	145	161	134	107	61.9	69.5	117
Belo Jardim	54.4	97.5	110	102	0	46.5	79.3	135	127	112	40.7	142	116	131	104	152	32	62	87.4
Gravatá	57.7	152	113	0	102	132	26.1	49.6	73.9	197	126	198	118	79.4	50.8	231	70	115	89.7
Santa Cruz do Capibaribe	60.5	157	0	113	110	140	88.5	106	63.2	205	134	206	57.4	93.6	113	239	77.7	122	23.2
Garanhuns	106	0	157	152	97.5	91.4	127	183	175	111	56.7	46.5	164	179	125	84.2	80	39.2	136
Caruaru	0	106	60.5	57.7	54.4	84.3	33.2	82.3	78.7	149	78.5	150	66.8	78.8	57.9	184	22.4	67	38.7
	Caruaru	Garanhuns	Santa Cruz do Capibaribe	Gravatá	Belo Jardim	Pesqueira	Bezerros	Limoeiro	Surubim	Buíque	São Bento do Una	Bom Conselho	Brejo da Madre de Deus	Bom Jardim	Bonito	Águas Belas	São Caitano	Lajedo	Toritama



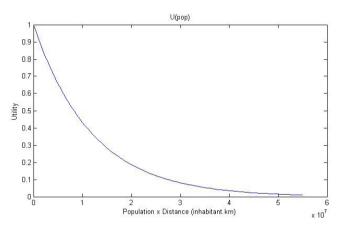


Figure 3 - One-dimensional utility function of population x distance

The solution of the one-dimensional utility function of population x distance, considering 6 medians is shown in Figure 4. The maximum value of $\sum_{i=1}^{ns} \sum_{j=1}^{ns} (U(pop)_{ij}) \cdot x_{ij}$ for nb = 6 is equal to 16.971. The substations chosen are {Caruaru, Garanhuns, Santa Cruz do Capibaribe, Pesqueira, Bezerros, Bom Jardim}.



Figure 4 - Solution of the one-dimensional utility function of population x distance

For the Health service indicator, the one-dimensional utility function of the population attribute is shown in Figure 5. The solution of the one-dimensional utility function of health service units x distance, considering 6 medians is shown in Figure 6.

The maximum value of $\sum_{i=1}^{ns} \sum_{j=1}^{ns} (U(hs)_{ij}) \cdot x_{ij}$ for nb = 6 is equal to 17.158. The substations

chosen are {Caruaru, Garanhuns, Santa Cruz do Capibaribe, Pesqueira, Bezerros, Surubim}. It is important to note that Surubim is the sixth substation included in the solution of the one-dimensional utility function of the health service attribute, and not Bom Jardim which came sixth in the one-dimensional utility function of the population attribute.

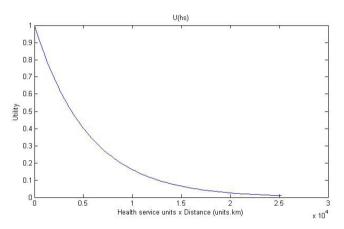


Figure 5 - One-dimensional utility function of health service units x distance



Figure 6 - Solution of the unidimensional utility function of health service units x distance

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As to the Industralization attribute, the one-dimensional utility function of industrialization attribute is shown in Figure 7. The solution of the one-dimensional utility function of industrialization (GDP) x distance, considering 6 medians is shown in Figure 8. The maximum value of $\sum_{i=1}^{ns} \sum_{j=1}^{ns} (U(ind)_{ij}) \cdot x_{ij}$ for nb = 6 is equal to 17.851. The substations chosen are {Caruaru, Garanhuns, Santa Cruz do Capibaribe, Belo Jardim, Bezerros, Limoeiro}. It is important to note that the substations in Belo Jardim and Limoeiro are part of the solution of the one-dimensional utility function of industrialization attribute, instead of Pesqueira and Surubim which were two of the six selected under the one-dimensional utility function of the health service attribute.

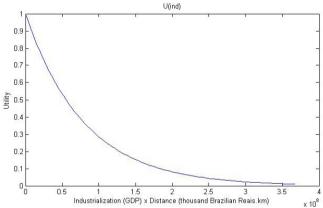


Figure 7 - One-dimensional utility function of industrialization (GDP) x distance



Figure 8 - Solution of the unidimensional utility function of industrialization (GDP) x distance



As the multi-attribute p-median model. the to shown in 9. The results are Figure maximum value of $\sum_{i=1}^{n} \sum_{j=1}^{n} (K_1 \cdot U(pop)_{ij} + K_2 \cdot U(ind)_{ij} + K_3 \cdot U(hs)_{ij} + K \cdot U(pop)_{ij} \cdot U(hs)_{ij}) \cdot x_{ij} \text{ for } nb = 0$ 6 is equal to 17.071. The constant scales represent the preferences of the manager of the current project. The parameters $K_1 = 0.2$, $K_2 = 0.5$, $K_3 = 0.2$, K = 0.1 were obtained through a sctrutured process in accordance with Keeney & Raiffa (1976). The substations chosen are {Caruaru, Garanhuns, Santa Cruz do Capibaribe, Pesqueira, Bezerros, Surubim}.



Figure 9 - Solution of the multi-attribute p-median model

The rosbustness of this model was verified when the sensitivity of the constant scales K_p , K_2 , K_3 and K was tested and no variation in the result was verified for variations of plus or minus 20%. We used the MIP-Solver of CPLEX 12.2 to solve our instances.

The case study shows that this solution to the problem of placing six backup transformers was very efficient, even with variations among the parameters, at indicating the appropriate places to locate the back-up transformers.

These solutions, in a situation where unplanned interruption occurs, would provide the best tradeoff between losses accompanying the dislocation of reserve equipment among the requisitioning substations and the adverse impacts arising relative to those aspects related to interrupting the supply of energy to the consumer.

Conclusion

The multi-attribute p-median model described in this paper evaluates alternatives of an unplanned interruption and seeks the best option among the losses



accompanying the dislocation of supply equipment among the substations affected, with adverse impact being regarded as whatever negative consequences arise from interruptions to the supply of energy. This model gives support to the decision maker about how to choose the most appropriate location, the one which can house the backup transformer at the distance most propitious to global gain, this being characterized by the best relationship among the available criteria for all the substations under consideration. CELPE considers that the location of reserve transformer units is of primary importance, because the limits placed on their resources restrict them from acquiring the volume of equipment needed for a "comfortable" margin of technical reserve equipment. Thus locating their existing reserves appropriately contributes significantly to maintaining adequate levels of service quality because of its effect on the availability of installations and, consequently, on there being an uninterrupted service, something which not only civil society increasingly demands, but also and, in particular, the regulatory agencies for energy distribution.

This study provides support for the decision-maker based on a structured model that brings together both the knowledge of client characteristics (population size, degree of industrialization, and index of health services) and the logistical aspects of the topology of the electrical supply system (distance and dislocation times between substations).

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